Simple scaling relations in geodynamics; the role of pressure in mantle convection

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Introduction

Pressure decreases interatomic distances in solids and this has a strong non-linear effect on such properties as thermal expansion, conductivity, and viscosity, all in the direction of making it difficult for small-scale thermal instabilities to form in deep planetary interiors. Convection is sluggish and large scale at high pressure. The Boussinesq approximation assumes that density, or volume (V), is a function of temperature (T) but that all other properties are independent of T, V and pressure (P), even those that are functions of V. This approximation, although thermodynamically (and algebraically) inconsistent, appears to be valid at low pressures, is widely used to analyze laboratory convection and is also widely used in geodynamics, including whole mantle convection simulations. Sometimes this approximation is supplemented with a depth dependent viscosity or with T dependence of parameters other than density.

It is preferable to use a thermodynamically self-consistent approach. To first order, the properties of solids depend on interatomic distances, or lattice volumetric strain, and to second order on what causes the strain (T, P, composition, crystal structure). This is the basis of Birch's Law [1], the seismic equation of state [2, 3], various laws of corresponding states and the quasi-harmonic approximation.

Scaling parameters are available for volume-dependent properties [3]. These can be written as volume derivatives:

Lattice thermal conductivity $d \ln K_L / d \ln V \sim 4$
Bulk modulus $d \ln K_T / d \ln V \sim 4$
Thermal expansivity $d \ln a / d \ln V \sim -3$
Viscosity $d \ln n / d \ln V \sim 40\text{-} 48$

Scaling to deep mantle conditions

The thermal boundary layer (TBL) at the boundary of a fluid cooled from above or heated from below grows as:

$$h \sim (k\, t)^{1/2}$$

where k is the thermal diffusivity, $K_L / p\, c_p$ and t is time.
The TBL becomes unstable, and detaches when the local Rayleigh number

$$Ra = a\, g\, (dT)\, h^3 / k\, n$$

exceeds about 1000 [4]. The new parameters are acceleration of gravity (g) and the temperature increase across the TBL (dT).

For parameters appropriate for the surface of the Earth the TBL becomes unstable at a thickness, h, of about 100 km [4], in good agreement with geophysical estimates of the thickness of the plates. The time-scale for this to happen is about 10^8 years, also in good agreement with the lifetime of surface oceanic plates.

The specific volume at the base of the mantle is 64% smaller than at the top [3]. The critical thickness of the lower TBL, ignoring radiative transfer, is therefore about 10 times larger than at the surface, or about 1000 km. If there is an appreciable radiative component, or if there is a chemical component to the density, then the scale lengths at the base of the mantle can be greater than this. In any case, the tomographic anomalies in the lower third of the mantle are very large [5], much larger than upper mantle slabs (cold plumes), consistent with the scaling theory.

The lifetime of the lower TBL, scaled from the upper mantle value, is $\sim 3 \times 10^{9}$ years. If radiation increases the thermal diffusivity by a factor of 8 this only reduces the timescale by a factor of 2. The surface TBL cools rapidly and becomes unstable quickly because of the magnitude of the thermal properties. The same theory, scaled for the density increase across the mantle, predicts large scale and long-lived features above the core.

Temperature

Temperature and pressure both affect the volume of a solid and it is volume that is the scaling parameter in the quasi-harmonic approximation and other equations of state. Pressure suppresses the effect of temperature on thermal expansion and, therefore, on all volume-dependent properties. Under lower mantle conditions P, composition and phase changes become the important controls on volume, buoyancy and seismic parameters. In general, T and P play opposing roles. One exception is the radiative part of the thermal conductivity. This increases as T^3 and possibly contributes to high conductivity of the deep mantle. Model calculations show that taking this into account can significantly affect the thermal history of the mantle and the style of mantle convection [6]. In the present context this is important since P and T combine to increase the importance of non-convective heat transfer and to suppress, or decrease the vigor of, mantle convection.

The low-spin transition in iron

Fe and Mg have similar ionic radii at low-pressure and substitute readily for each other in upper mantle minerals. Fe is more-or-less uniformly partitioned among the major minerals. This and low temperatures suppress the role of radiative transport of heat. Fe undergoes a spin-transition at high pressure with a large reduction in ionic radius [7, 8]. The major minerals in the deep mantle are predicted to be almost Fe-free perovskite [$MgSiO_3$] and Fe-rich magnesio-wustite [(Mg,Fe)O]. This has several important geodynamic implications. Perovskite, being the major phase, will control the conductivity and viscosity. Radiative conductivity is expected to be high in Fe-poor minerals and viscosity is expected to be low [8]. Both effects will tend to stabilize the mantle against convection and decrease the Rayleigh number. Over time, the dense magnesio-wustite may accumulate, irreversibly, at the base of the mantle and, in addition, may interact with the core. The lattice conductivity of this iron-rich layer will be high and the radiative term should be low but the trade-offs are unknown. A thin layer convects sluggishly (because of the h^3 term in the Rayleigh number) but its presence slows down the cooling of the mantle and the core. The overlying FeO-poor layer may have high radiative conductivity, because of high T and transparency, and high viscosity and low expansivity, because of P effects on volume. This part of the mantle will also convect sluggishly. If it represents about one-third of the mantle (by depth) it will have a Rayleigh number about 30 times less than Rayleigh numbers based on whole mantle convection and orders of magnitude less than Ra based on P=0 properties.

The low thermal expansivity at high pressure means that moderate jumps in intrinsic density between layers in the mantle can permanently stabilize chemical layering [9, 10, 11]. Unreasonably high lower mantle temperatures do not occur since most of the radioactivity is in the crust and upper mantle [3].

Summary

Pressure, sphericity and the distribution of radioactive elements in the mantle, breaks the symmetry between the surface and lower TBLs of the mantle. The surface TBL is responsible for plate tectonics and for organizing convective motions in the upper mantle [4, 5, 12]. The lower TBL heats slowly since only a small amount of heating is available, either from the mantle or the core [3]. This contributes to the sluggishness of deep mantle convection. The slow heating and the low thermal buoyancy requires that enormous features must develop to carry away any heat not conducted or radiated away. This is in marked contrast to conditions at the surface. Pressure also contributes to the chemical stratification of the mantle and the inability of temperature to overcome intrinsic density contrasts [9]. At high pressure, temperature has little effect on density and other physical properties. The exception is radiative conductivity which reinforces the P effect on lattice conductivity in the direction of suppressing small-scale mantle convection in the deep mantle. Mantle convection is evidently characterized by narrow downwellings and broad diffuse upwellings [14], the opposite of the plume model [13] but consistent with plate tectonics and mantle tomography [5].

References and Notes

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